Solution of variational Problems using New Iterative Method

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Abstract: I used New Iterative Method (NIM) on the variational problems. This is recently developed method which is very easy and efficient developed by Daftardar Gejji and Hossein Jafri [1].In this study the variational problems are solved to check the ability of this method for solving non linear and linear ordinary differential equations. The results obtained are very useful and close to the exact solution.

Keywords: New iterative method; Eigen value problems;

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1. Introduction

In the problem of geodesics we want to determine the line of minimum length connecting two given points on a certain surface. This problem was solved in 1698 by Jacob Bernoulli and a general method for solving such problems was given in the works of Euler and Lagrange.

In the isoperimetric problem, it is required to find a closed line of given length l bounding a maximum area S. The solution of this problem is circle. General methods for solving problems with isoperimetric conditions were elaborated by Euler.

In view of the central importance of the variational problems for so many fields of pure and applied mathematics, much thought has been devoted to the designing of efficient methods to solve the variational problems. We find the solutions of variational problems by using the new iterative method. The results are compared with those obtained by the numerical methods available in the literature to establish the efficiency of the method.

2. The New Iterative Method

Consider the following general functional equation

$$y(\bar{x}) = f(\overline{x}) + N\big(y(\bar{x})\big), \qquad (1)$$

Where N is nonlinear from a Banach space B→B, f is a known function and $\bar{x} = (x_1, x_2, \ldots \ldots, x_n)$.

We are looking for a solution y of eq. (1) having the series form

$$y(\bar{x}) = \sum_{n=0}^{\infty} y_i (\bar{x}) \qquad (2)$$

The nonlinear operator N can be decomposed as

$$N(\sum_{n=0}^{\infty} y_i) = N(y_0) + \sum_{i=1}^{\infty}\{N(\sum_{j=0}^{i} y_j) - N(\sum_{j=0}^{i-1} y_j)\} \qquad (3)$$

From Equations (2) and (3), Eq. (1) is equivalent to

$$\sum_{i=0}^{\infty} y_i = f + N(y_0) + \sum_{i=1}^{\infty} \{N(\sum_{j=0}^{i} y_j) - N(\sum_{j=0}^{i-1} y_j)\}. \tag{4}$$

We define the recurrence relation

$$y_0 = f,$$

$$y_1 = N(y_0), \tag{5}$$

$$y_{m+1} = N(y_0 + \cdots + y_m) - N(y_0 + \cdots + y_{m-1})$$

Then

$$(y_1 + \cdots + y_{m+1}) = N(y_0 + \cdots + y_m),$$

and

$$\sum_{i=0}^{\infty} y_i = f + N(\sum_{i=0}^{\infty} y_i). \tag{6}$$

The k-term approximation solution of Eq. (1) is given by $y = y_0 + y_1 + \cdots + y_{k-1}$

If N contracts i.e.$\| N(x) - N(y) \| \le k \| x - y \|, 0 < k < 1,$ then

$$\| y_{m+1} \| = \| N(y_0 + \cdots + y s_m \| - N \| y_0 + \cdots + y_{m-1} \| \le k \| y_m \| \le k^m \| y_0 \|,$$

m =0, 1, 2……

And series $\sum_{i=0}^{\infty} y_i$ uniformly and absolutely converges to solution of equation (1). A unique solution, with respect to Banach fixed point theorem [15].

3. The applications of New Iterative Method on the Variational problems

Example .1 Consider the following variational problem:

$$min\ v = \int_0^1 (y(x) + y'(x) - 4e^{3x})^2 dx,$$

With given boundary conditions $y(0) = 1, \qquad y(1) = e^3,$

By using the L-L Lagrange equation, The corresponding Euler-Lagrange equation is

$$y'' = y + 8e^{3x}$$

The corresponding integral equation is

$$y(x) = 1 + Ax + \int_0^x \int_0^x (y + 8e^{3x}) dx dx$$

Setting $y_0 = 1 + Ax$ $N(y) = \int_0^x \int_0^x (y + 8e^{3x}) dx dx$

Applying the algorithm of new iterative method

$$y_1 = N(y_0) = (1/6)Ax^3 + (1/2)x^2 - (8/3)x + (8/9)e^{3x} - (8/9)$$

$$\begin{aligned} y_2 = N(y_0 + y_1) - N(y_0) \\ = (1/(120))Ax^5 + (1/(24))x^4 - (4/9)x^3 - (4/9)x^2 - (8/(27))x + (8/(81))e^{3x} \\ - (8/(81)) \end{aligned}$$

$$\begin{aligned} y_3 = N(y_0 + y_1 + y_2) - N(y_0 + y_1) \\ = (1/(5040))Ax^7 + (1/(720))x^6 - (1/(45))x^5 - (1/(27))x^4 - (4/(81))x^3 \\ - (4/(81))\,x^2 - (8/(243))x + (8/(729))e^{3x} - (8/(729)) \end{aligned}$$

$$\begin{aligned} y_4 = N(y^0 + y^1 + y^2 + y^3) - N(y^0 + y^1 + y^2) \\ = (1/(362880))Ax^9 + (1/(40320))x^8 - (1/(1890))x^7 - (1/(810))x^6 \\ - (1/(405))x^5 - (1/(243))x^4 - (4/(729))x^3 - (4/(729))x^2 - (8/(2187))x \\ + (8/(6561))\,e^{3x} - (8/(6561)) \end{aligned}$$

$$\begin{aligned} y(x) = y_0 + y_1 + y_2 + y_3 + y_4 \\ = ((6560)/(6561))e^{3x} - ((6560)/(2187))x + (1/6)Ax^3 + (1/(120))Ax^5 \\ + (1/(5040))\,Ax^7 + (1/(362880))Ax^9 + Ax + (1/(1458))x^2 - ((364)/(729))x^3 \\ + (1/(1944))\,x^4 - (2/(81))x^5 + (1/(6480))x^6 - (1/(1890))x^7 + (1/(40320))x^8 \\ + (1/(6561)) \end{aligned}$$

$$\begin{aligned} y(1) = ((426457)/(362880))A + ((6560)/(6561))e^3 - ((103539167)/(29393280)) \\ = 1.1752A + 16.560 \end{aligned}$$

$$1.1752A + 16.560 = 20.086$$

$$1.1752A = 3.526$$

$$A = 3.0003$$

By putting the value of constant A in $y(x)$ so,

$$\begin{aligned} y(x) = 8.268 \times 10^{-6}x^9 + \left(\frac{1}{40320}\right) x^8 + 6.6197 \times 10^{-5}x^7 + \left(\frac{1}{6480}\right) x^6 + 3.1114 \times 10^{-4}x^5 \\ + \left(\frac{1}{1944}\right) x^4 + 7.3587 \times 10^{-4}x^3 + \left(\frac{1}{1458}\right) x^2 + 7.5725 \times 10^{-4} \end{aligned}$$

In this example it is clear the new iterative method can be consider as an efficient method.

Example .2 Consider the following variational problem:

$$min\ v = \int_0^{\frac{\pi}{2}} (y''2 - y^2 + x^2) dx$$

That satisfies the conditions $y(0) = 1$, $y'(0) = 0$, $y\left(\frac{\pi}{2}\right) = 0$, $y'\left(\frac{\pi}{2}\right) = -1$,

The corresponding Euler-Lagrange equation is

$$y^4 - y = 0,$$

The corresponding integral equation is

$$y(x) = 1 + ((Bx^2)/2) + ((Ax^3)/6) + \int_0^x \int_0^x \int_0^x \int_0^x y \, dx dx dx dx$$

Setting $y_0 = 1 + ((Bx^2)/2) + ((Ax^3)/6)$ $N(y)) = \int_0^x \int_0^x \int_0^x \int_0^x y \, dx dx dx dx$

Applying the algorithm of new iterative method

$$y_1 = N(y_0) = (1/(5040))Ax^7 + (1/(720))Bx^6 + (1/(24))x^4$$

$$y_2 = N(y_0 + y_1) - N(y_0) = (1/(5040))Ax^7 + (1/(720))Bx^6 + (1/(24))x^4$$

$$y_3 = N(y_0 + y_1 + y_2) - N(y_0 + y_1)$$
$$= (1/(1307\ 674368000))Ax^{15} + (1/(87\ 178291200))B\ x^{14} + (1/(479001600))x^{12}$$

$$y_4 = N(y^0 + y^1 + y^2 + y^3) - N(y^0 + y^1 + y^2)$$
$$= (1/(121645100\ 408832000))Ax^{19} + (1/(6402373\ 705728000))\ Bx^{18}$$
$$+ (1/(20922\ 789888000))x^{16}$$

$$y(x) = y_0 + y_1 + y_2 + y_3 + y_4$$
$$= (1/(121645100\ 408832000))Ax^{19} + (1/(6402373\ 705728000))\ Bx^{18}$$
$$+ (1/(20922\ 789888000))x^{16} + (1/(1307\ 674368000))\ Ax^{15}$$
$$+ (1/(87\ 178291200))Bx^{14} + (1/(479001600))x^{12} + (1/(39916800))Ax^{11}$$
$$+ (1/(3628800))Bx^{10} + (1/(40320))x^8 + (1/(5040))Ax^7 + (1/(720))Bx^6$$
$$+ (1/(24))x^4 + (1/6)Ax^3 + (1/2)Bx^2 + 1$$

$$y((\pi/2)) = (1/(48))\pi^3 A + (1/8)\pi^2 B + (1/(645120))\pi^7 A + (1/(46080))\pi^6 B$$
$$+ (1/(81\ 749606400))\pi^{11}A + (1/(3715891200))\pi^{10}B$$
$$+ (1/(42849873\ 690624000))\pi^{15}A + (1/(1428329\ 123020800))\ \pi^{14}B$$
$$+ (1/(63777\ 066403145\ 711616000))\pi^{19}\ A$$
$$+ (1/(1678\ 343852714\ 360832000))\pi^{18}B + (1/(384))\ \pi^4 + (1/(10321920))\pi^8$$
$$+ (1/(1961\ 990553600))\pi^1 + (1/(1371195958\ 099968000))\pi^{16} + 1$$

Putt $y\left(\left(\frac{\pi}{2}\right)\right) = 0$ is boundary condition

$$0.65065A + 1.2546\ B + 1.2546 = 0$$

By using B.Cs $y'\left(\frac{\pi}{2}\right) = -1$,

$$A = -1.2546, B = -1.6506$$

The series form of the solution is

$$y(x) = -2.0367 \times 10^{-21}x^{19} - 1.5617 \times 10^{-16}\,x^{18} + (1/(20922\,789888000))x^{16} - 1.8946$$
$$\times 10^{-16}\,x^{15} - 1.1469 \times 10^{-11}x^{14} + (1/(479001600))x^{12} - 6.2067 \times 10^{-12}\,x^{11}$$
$$- 2.7554 \times 10^{-7}x^{10} + (1/(40320))x^{8} - 4.9157 \times 10^{-8}\,x^{7} - 1.3887 \times 10^{-3}x^{6}$$
$$+ (1/(24))\,x^{4} - 4.1292 \times 10^{-5}x^{3} - 0.49994\,x^{2} + 1$$

Using more components of y(x) we can found better results.

4. Conclusion

In this paper I have discussed the variational problems using recently developed new iterative method developed by Versha Daftardar Gejji and Hossein jafari. The results obtained are very close to the exact solution and the calculations are also reduced. Using few iterations we obtained the desired results. These results show the efficiency and effectiveness of the new iterative method.

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